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Modeling Large Scale Troop Movement Using Reaction Diffusion Equations

MaryAnne Fields

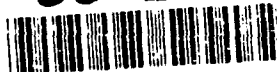
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CONTENTS

	<u>Page</u>
LIST OF FIGURES	v
1. INTRODUCTION	1
2. MOVEMENT ON SMOOTH SURFACES	2
3. MOVEMENT ON ROADS	6
4. RESPONSIVE MOVEMENT	11
5. CONCLUSIONS	20
6. FUTURE WORK	22
7. REFERENCES	23

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LIST OF FIGURES

	<u>Page</u>
Figure 1. A Smooth Surface with Four Hills	3
Figure 2. A Viscous Fluid Moving on the Smooth Surface Given in Figure 1	5
Figure 3. A Narrow Pass	7
Figure 4. An Army Moving Through the Narrow Pass Shown in Figure 3	8
Figure 5. Four Vector Fields Attracting the Army to the Path $P(x) = x^2$	10
Figure 6. An Army Moving on the Path $p(x) = x^2$ with a large σ	12
Figure 7. An Army Moving on the Path $p(x) = x^2$ with a small σ	13
Figure 8. Uncontrolled Attraction of Blue Army to Red Army	15
Figure 9. Controlled Attraction of Blue Army to Red Army	17
Figure 10. A Battle Between the Red and Blue Armies with No Response from Red	18
Figure 11. A Battle Between the Red and Blue Armies Allowing Red to Retreat	19
Figure 12. A Two-Sided Battle Between the Red and Blue Armies	21

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1. INTRODUCTION

Many of the current battlefield simulation models are storage and time intensive and require detailed game piece descriptions, which must be updated periodically during the simulation. Since game pieces on a battlefield tend to move and act as groups rather than as individuals, an alternate approach to battlefield simulation is to consider large numbers of similar units in aggregates and to model the behavior of the aggregates rather than the individual pieces. The behavior of these aggregates may be described by reaction diffusion equations (RDEs) normally used to describe the movement, spread, and interaction of biological or chemical species.

Let \mathbf{D} be a region of \mathbf{R}^2 representing the battlefield and B and R be two battlefield species (homogeneous forces of battalion size or higher). A general RDE describing the interaction of B and R is given by the following system of equations:

$$b_t = D_{B_x} b_{xx} + D_{B_y} b_{yy} + V_{B_x} b_x + V_{B_y} b_y + I_B(b,r), \quad (1a)$$

$$r_t = D_{R_x} r_{xx} + D_{R_y} r_{yy} + V_{R_x} r_x + V_{R_y} r_y + I_R(b,r), \quad (1b)$$

in which:

b, r - density of species B and R, respectively;

$V_{B_x}, V_{B_y}, V_{R_x}, V_{R_y}$ - velocity coefficients;

$D_{B_x}, D_{B_y}, D_{R_x}, D_{R_y}$ - diffusion coefficients;

I_B, I_R - interaction functions.

In the general form shown above, all the coefficients given in these equations are functions of space, time, and the dependent variables themselves. For a comprehensive review of RDEs, the reader is referred to the works of Aris (1975), Fife (1979), and Murray (1989). There have been some preliminary applications of RDEs to battlefield simulation. The reader is referred to the works of Protopopescu et al. (1989), Santoro et al. (1989), and Azmy (1991) for more details of these models.

The intent of this report is to lay the groundwork for RDE battlefield simulations, rather than use the RDEs in a specific simulation. For this purpose, we will assume that \mathbf{D} is the unit square with boundaries, $x = 0$, $y = 0$, $x = 1$, and $y = 1$. There is no

loss of generality since we can scale \mathbf{D} to any size battlefield. To use the RDE format most advantageously, we will use continuous velocity and diffusion coefficient functions wherever possible. The values for the velocity and diffusion coefficient functions, as well as the initial distributions for \mathbf{B} and \mathbf{R} , are to be chosen to illustrate specific points in the report rather than to simulate real engagements. The boundary conditions are reflecting or absorbing, depending on the situation being modeled.

We concentrate on developing velocity coefficients to describe realistic movement. The diffusion coefficients, which measure the natural tendency for species to spread, are set to a constant value. The simplest model for interactions between the two armies assumes that the attrition for either side is proportional to the number of possible "encounters" between individuals of different armies in a specific section of the battlefield. The attrition for either side is modeled as the product of the densities of the two species and a constant of proportionality. In those examples for which they are needed, the interaction terms are of this form. Note that the term "army" is used in this report to denote a homogeneous armed force of battalion size or larger.

Three types of movement are discussed: movement on smooth surfaces, movement along predefined paths, and "responsive" movement. In each type of movement, the battlefield is treated as a velocity vector field similar to an electric or magnetic field. To model movement on smooth surfaces, the velocity vector associated with each point on the battlefield depends on the slope or some other local characteristic of the terrain. To simulate the movement of troops along a path P , the velocity vector at any point (x,y) of the battlefield depends on the distance between the path P and that point. The "responsive" movement model, which simulates reactions to battlefield conditions such as the presence of enemy troops or heavy friendly losses, involves a dynamic modification of the velocity vector field using the concentration of either enemy or friendly forces.

2. MOVEMENT ON SMOOTH SURFACES

A physical analogy of the movement of forces on the battlefield is the flow of a small quantity of viscous material, B , over a nonuniform surface. To illustrate this analogy, suppose that $f: \mathbf{R}^2 \rightarrow \mathbf{R}$, the surface shown in Figure 1, represents a section of a

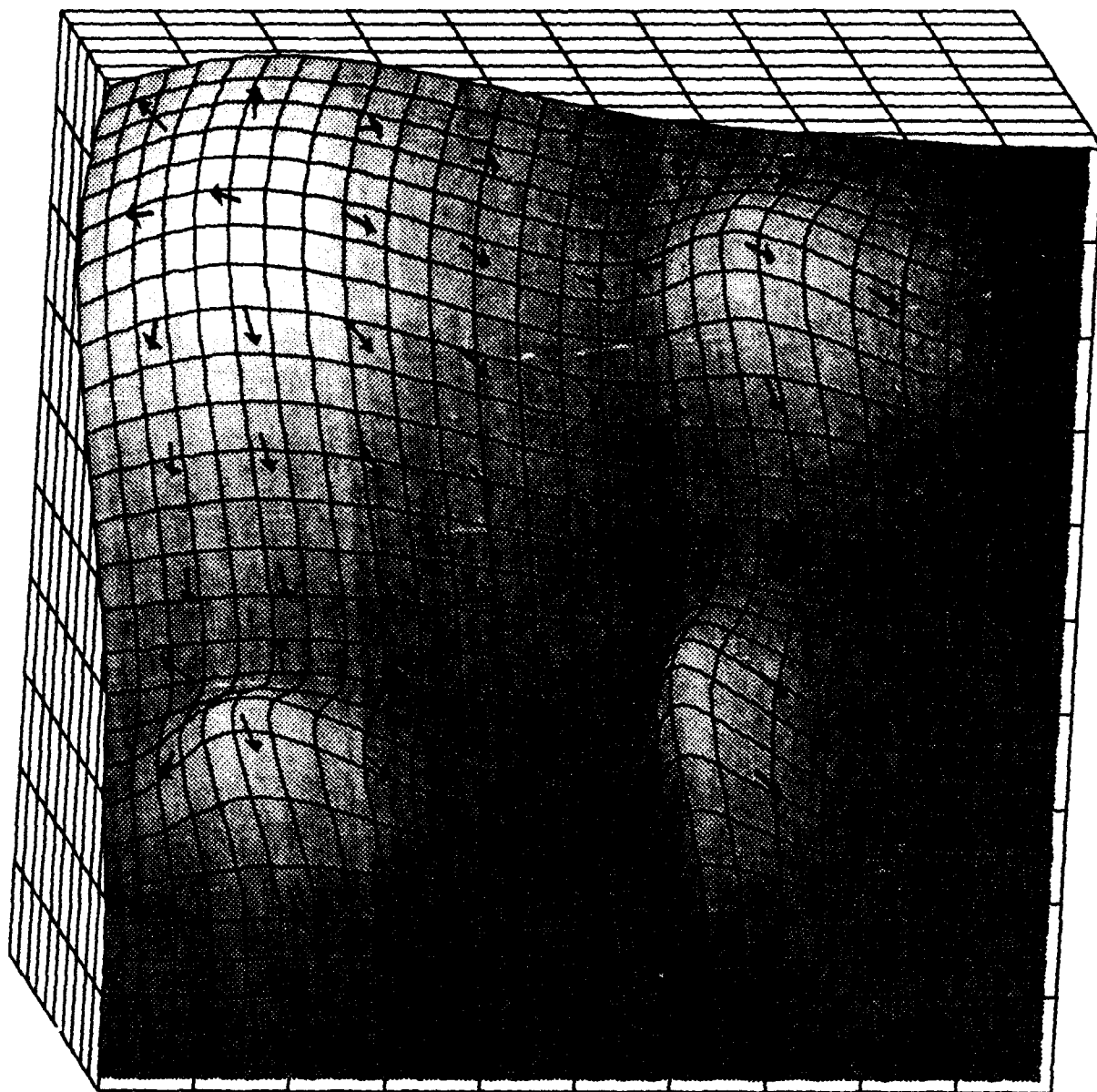


Figure 1. A Smooth Surface with Four Hills.

battlefield. The surface consists of one large hill at the upper left of the figure and three smaller hills. Fluid released at some point on the surface would flow downhill and pool in the vicinity of the local minima of the surface. If, as in Figure 1, there are no local minima, the material will flow downhill until it reaches a boundary for the region **D**. More precisely, at any point (x,y,z) , on the surface f , an element of fluid would flow in the direction of the negative gradient, $-\nabla f = -(f_x, f_y)$. The arrows shown in Figure 1 show the negative gradient vector at several points on the surface.

An equation that describes the movement of the material B over the surface is

$$b_t = Db_{xx} + Db_{yy} - f_x b_x - f_y b_y, \quad (2)$$

in which b is the density function for the material, D is a constant diffusion coefficient and f is the equation of the terrain surface. The boundary conditions are absorbing.

Figure 2 is a sequence of snapshots showing the density function, b , at different times as the material B moves from the top of the hill in the upper left-hand corner of Figure 1 to the lower regions of the surface. The density of B is indicated by the shades of grey in the snapshots with the darker grey indicating the higher density. The surface is shown as a contour map. In the earlier snapshots, most of the material is on the hill at the upper left-hand corner moving down the large hill. As the simulation progresses, the material separates, going around instead of over the lower hills. Examining Equation 1, notice that the gradient of the surface determines the dominant direction of motion at any point on the surface, while the diffusion coefficient determines the spread of the material relative to that direction. In this simulation, the material B does not climb the lower hills because the diffusion coefficient is small relative to the size of the velocity vector at the base of these hills.

This same model could be used to model the movement of an army across a section of terrain. The army must be homogeneous with respect to its movement characteristics, since Equation 1 does not track the movements of individual units or soldiers. The model can be easily extended to model the movements of several distinct groups within a heterogeneous force by adding more equations.

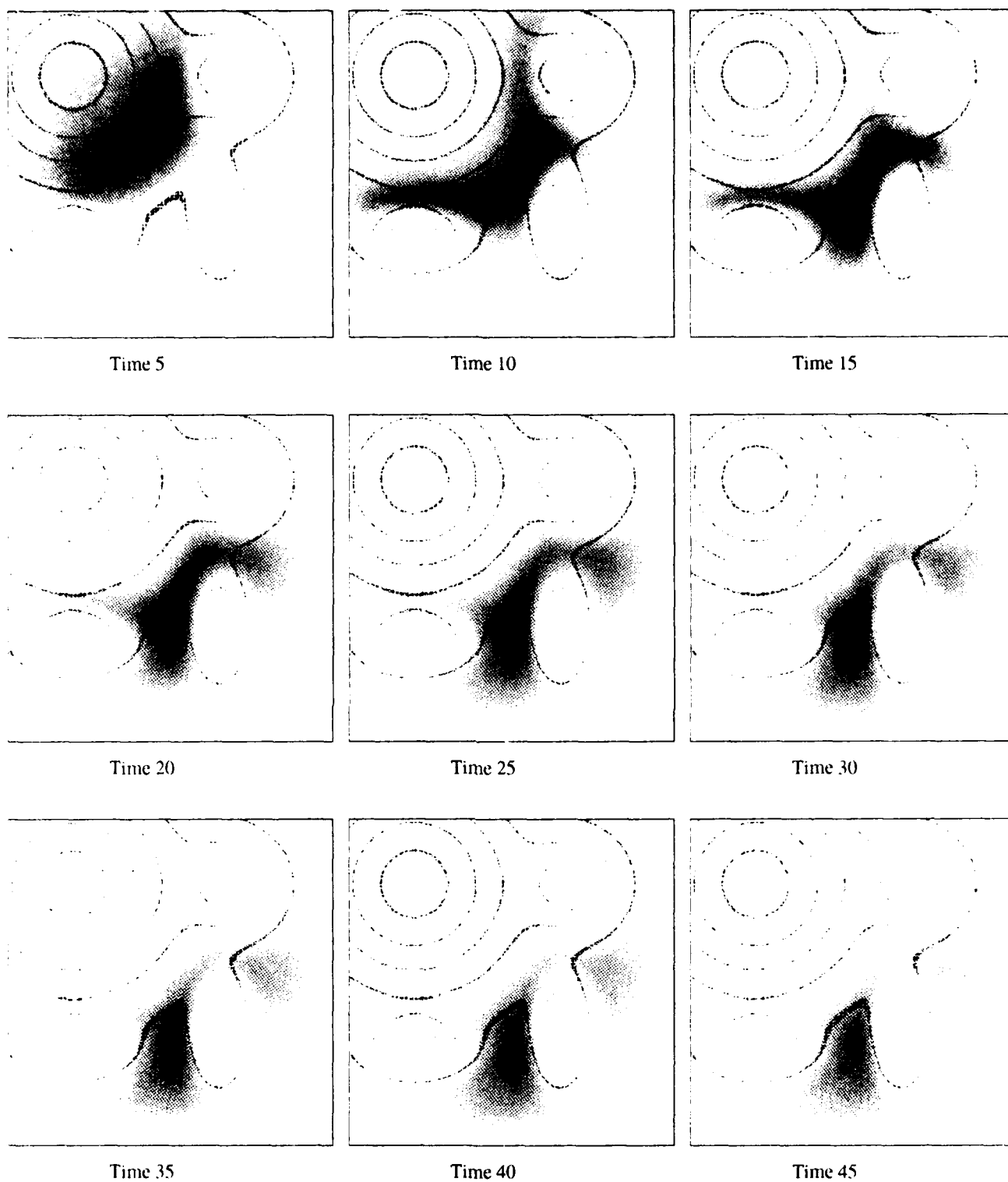


Figure 2. A Viscous Fluid Moving on the Smooth Surface Given in Figure 1.

In the next example, suppose that B is a homogeneous army moving through the narrow pass shown in Figure 3. For this scenario, the army starts at the upper left-hand corner of the surface, on the floor of the valley just before entering the pass, then proceeds through the pass to the lower right-hand corner. The basic model equation is the same as the previous example, given by Equation 1. Only the surface f changes.

Figure 4 shows a sequence of snapshots of the army's progress through the pass, taken at various times, t_k . As in Figure 2, the grey area of the pictures corresponds to the density function, $b(x, y, t_k)$, of the army with the darker grey indicating the higher density. Some contours of the pass are also shown in the pictures for reference. In the first snapshot, the shape of the density function is roughly that of a bivariate normal distribution - almost symmetrical with the highest concentration near the center of the distribution. At this point, before the army enters the pass, diffusion is the dominant effect. In the next two snapshots, the constraints of the pass force the army into an elongated shape. Notice that one of the advantages of using the negative gradient vector field to control movement is that we can simulate one-way "boundary" conditions on the interior of the region D . Units within the pass cannot climb the sides to leave; they must exit the pass at the bottom right.

Since most movement on the battlefield is directed rather than simply dictated by the terrain, the limitations of this model become quickly apparent. However, it provides background and a basis of discussion for the models developed in the remainder of the paper. The idea of fluid flowing on a surface will be expanded to model directed movement on the battlefield by altering the underlying surface to "force" the armies to move in the desired fashion.

3. MOVEMENT ON ROADS

Armies are frequently required to move along roads or predetermined paths to specific destinations. The two examples used in the previous section suggest that armies can be forced into specific regions of the battlefield by using properties of the topographical surface of the battlefield. In this section, we use the same qualitative behavior observed in the previous section, attracting the armies to the roads and other man-made features of the battlefield rather than to topological features of the surface. Essentially, we want

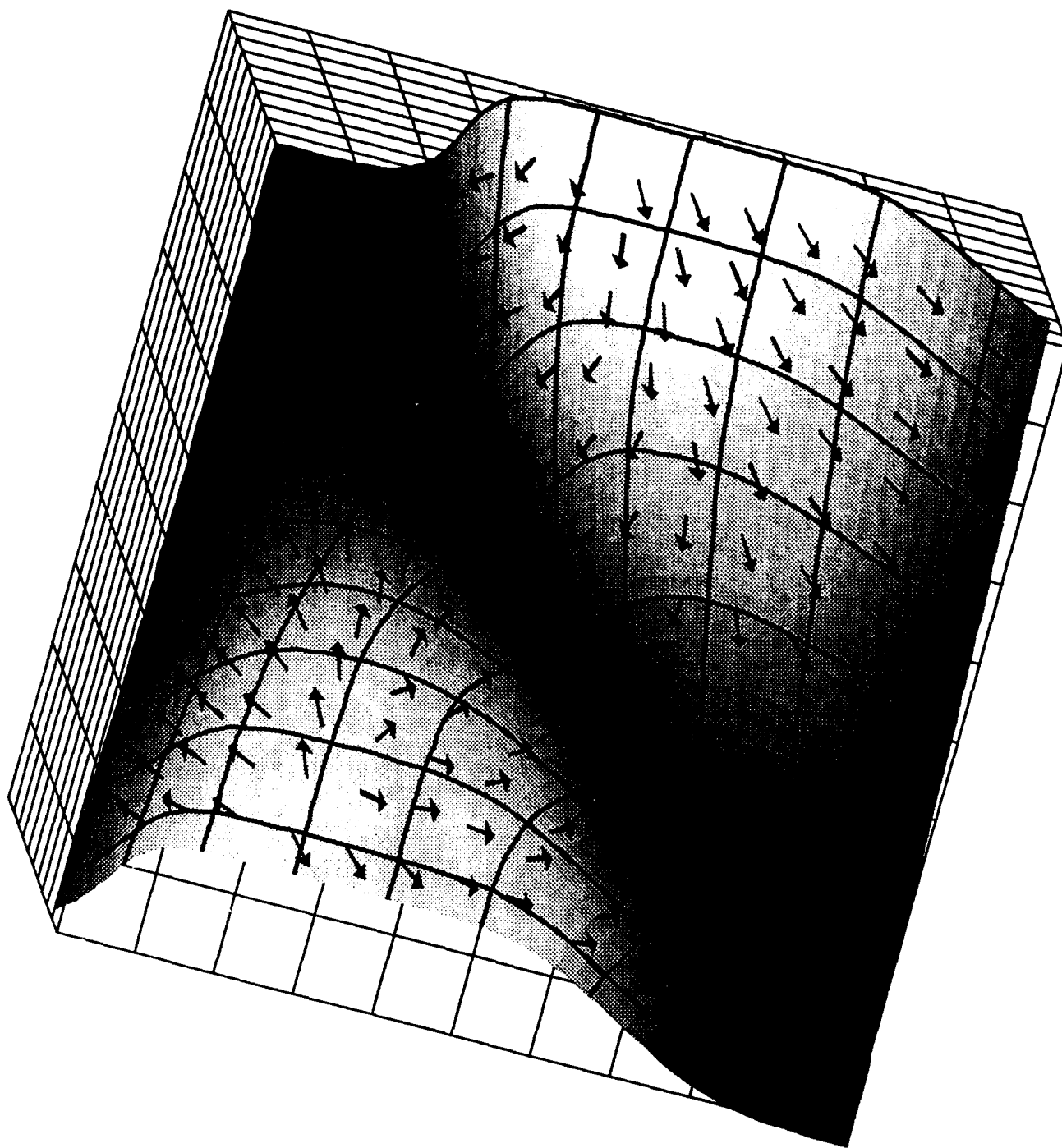


Figure 3. A Narrow Pass.

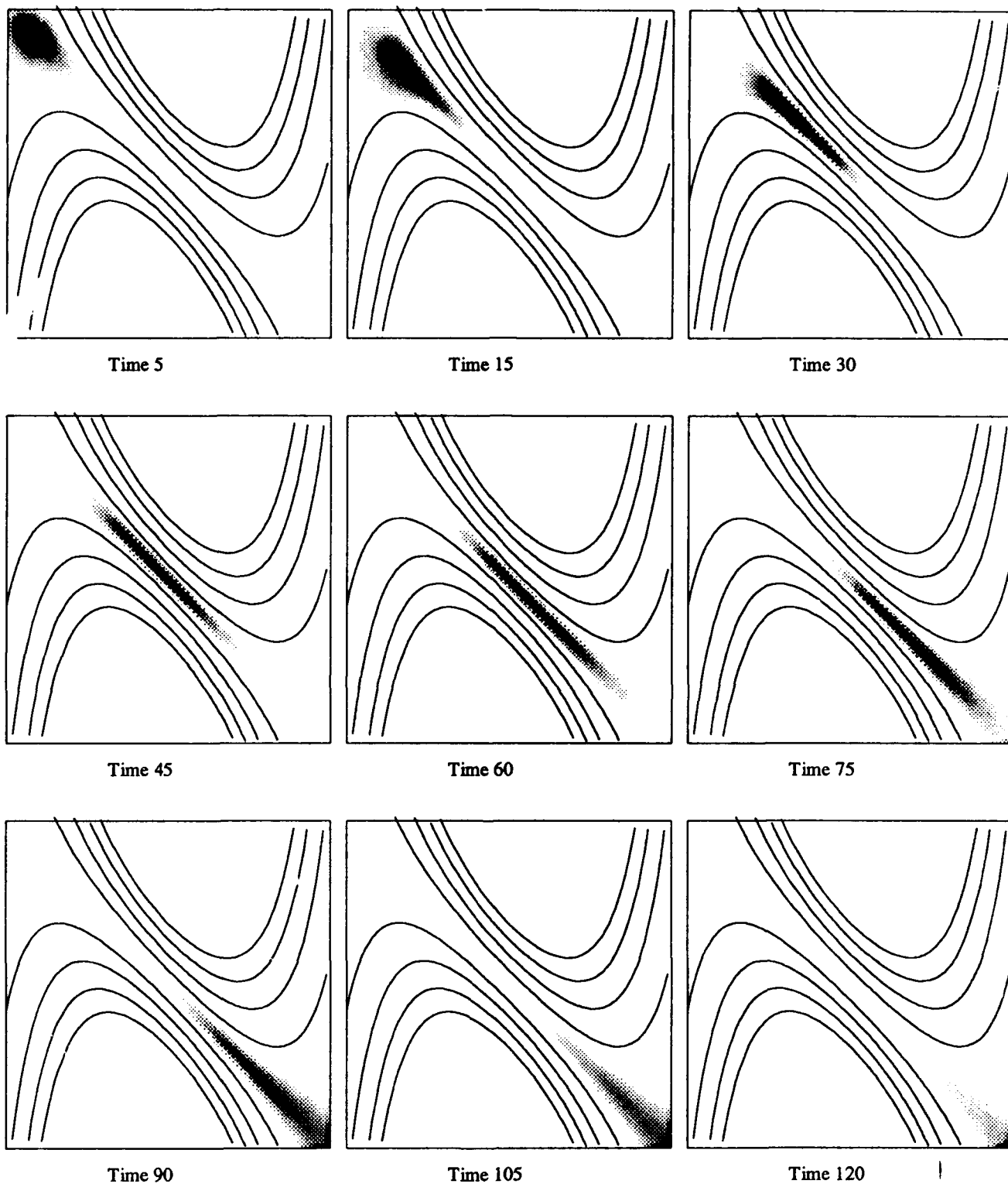


Figure 4. An Army Moving Through the Narrow Pass Shown in Figure 3.

to construct a function, similar to the function that describes the narrow pass in the previous example, that forces the army into a small region of the battlefield where it moves to some final destination.

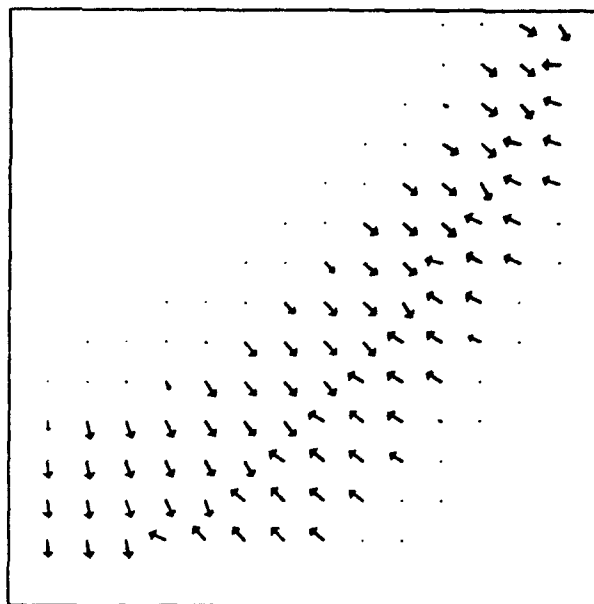
As in the previous section, let \mathbf{D} be a region of \mathbf{R}^2 representing the battlefield. For simplicity, we will assume that the terrain is flat so that units can move freely about the battlefield. Let p be a function of the real numbers which, when its graph is restricted to \mathbf{D} , describes the path that the army follows to its destination $q = (x_0, y_0) \in \mathbf{D}$. We want to construct a function of two dimensions, f , with the associated vector field $\nabla f = (f_x, f_y)$ so that units following these gradient vectors will move toward the path, p , then follow the path to the final destination, q . The function, f , satisfies the following set of conditions:

1. For any point on the path, $(x, p(x))$, the surface is higher than for nearby points not on the road.
2. The surface f has no local minimum.
3. The global maximum for the surface f over the region \mathbf{D} is at the point, $q = (x_0, y_0 = p(x_0))$.
4. The surface f is sufficiently smooth.

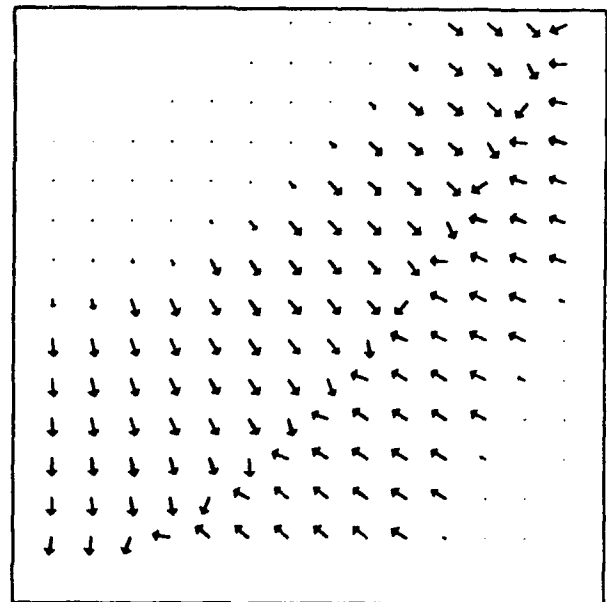
A family of functions that satisfy these criteria is given by

$$f(x, y) = g(x) e^{-\sigma(y - p(x))^2}, \quad (3)$$

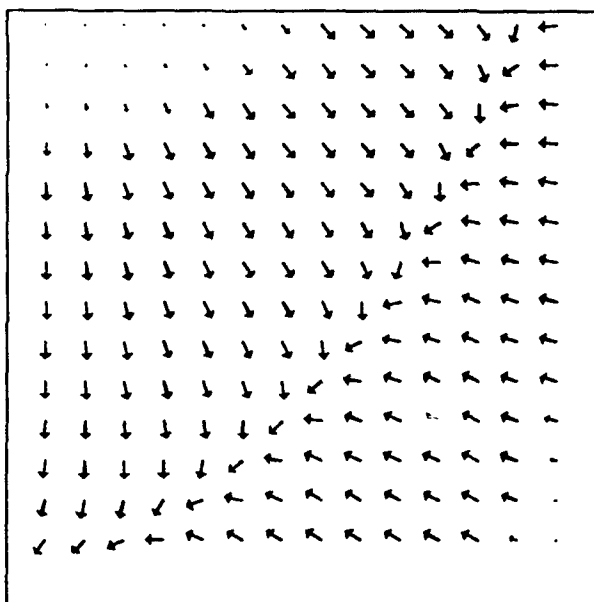
with $\sigma > 0$ and $g(x)$ a continuous function over \mathbf{D} with a maximum at (x_0, y_0) . The best choice for $g(x)$ is a function of the arc length of the path, p . However, if $|p''|$ is relatively small for all $0 \leq x \leq 1$, other functions may be used. Exponential functions work nicely since they are infinitely differentiable (provided the path, p , is sufficiently smooth), and the parameter σ can be varied to control the strength of attraction to the path. Exponential functions are also very easy to visualize. Figure 5 shows some examples of the vector fields generated by Equation 3. For these examples, \mathbf{D} is the unit square, the path is $p(x) = x^2$, and $g(x) = 1 - x$, (forcing units to travel to the origin); only σ has been varied.



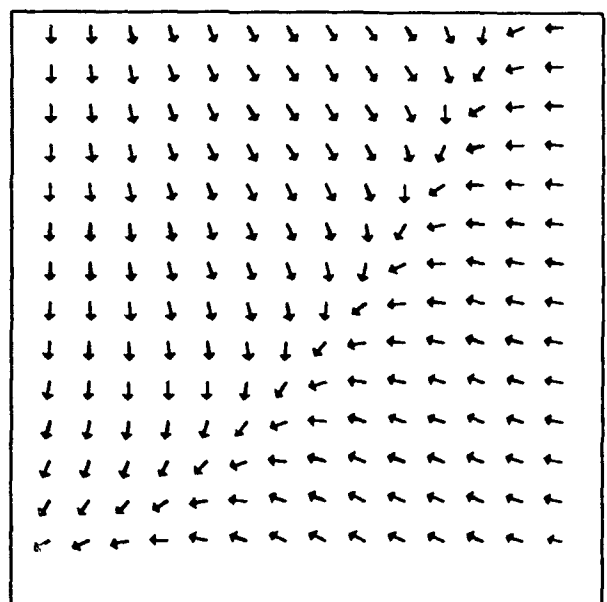
$\sigma = 100$



$\sigma = 30$



$\sigma = 10$



$\sigma = 5$

Figure 5. Four Vector Fields Attracting the Army to the Path $p(x) = x^2$.

The length of the arrows shown in these fields is proportional to the magnitude of the gradient at these points; regions without arrows indicate that the gradient is near zero for these regions. The basic equation for modeling armies moving along the path p is the same as in the previous section:

$$b_t = Db_{xx} + Db_{yy} + f_x b_x + f_y b_y. \quad (4)$$

D remains constant; f_x and f_y are determined from Equation 3. In this example, two units of a homogeneous army have been ordered to follow the road, $p(x)=x^2$, to the lower left-hand corner of the battlefield. To demonstrate that the road "attracts" nearby army units, neither unit is positioned on the road at the start of the simulation. Figures 6 and 7 show a sequence of snapshots of an army moving on a road. The effect of decreasing the size of σ can be seen by comparing the two sets of snapshots. In Figure 6, in which σ is large, the army follows the road very tightly. In Figure 7, the army is spread out farther. However, it takes longer for the entire army to reach the road in Figure 6 than it does in Figure 7. This can be explained by examining the corresponding vector fields given in Figure 5. For $\sigma = 30$, there are "dead" zones on the battlefield where the magnitude of the velocity vector is very small. Units in these areas move slowly toward the road if at all. As σ increases, the area of these dead zones increases to the point that only units very close to the road are attracted to it.

The velocity of the army in this model is roughly $|\nabla f|$ for units close to the path. This may not be desirable since homogeneous units of an army usually move at about the same speed. It is possible to modify the velocity coefficient function so that variations in speed can be controlled, but this will involve the use of either step functions or rational functions of the exponential functions already used in the velocity terms. As a practical matter, the speed of units can be easily controlled in the numerical solution of the model equations by modifying the velocity vector field before it is used.

4. RESPONSIVE MOVEMENT

Movement on the battlefield depends as much on changing battlefield conditions as on the initial plan that specifies routes and objectives for each army. In fact, the initial plan may include a number of contingency plans to handle situations such as unexpected encounters with enemy units. Responsive movement will be defined as any

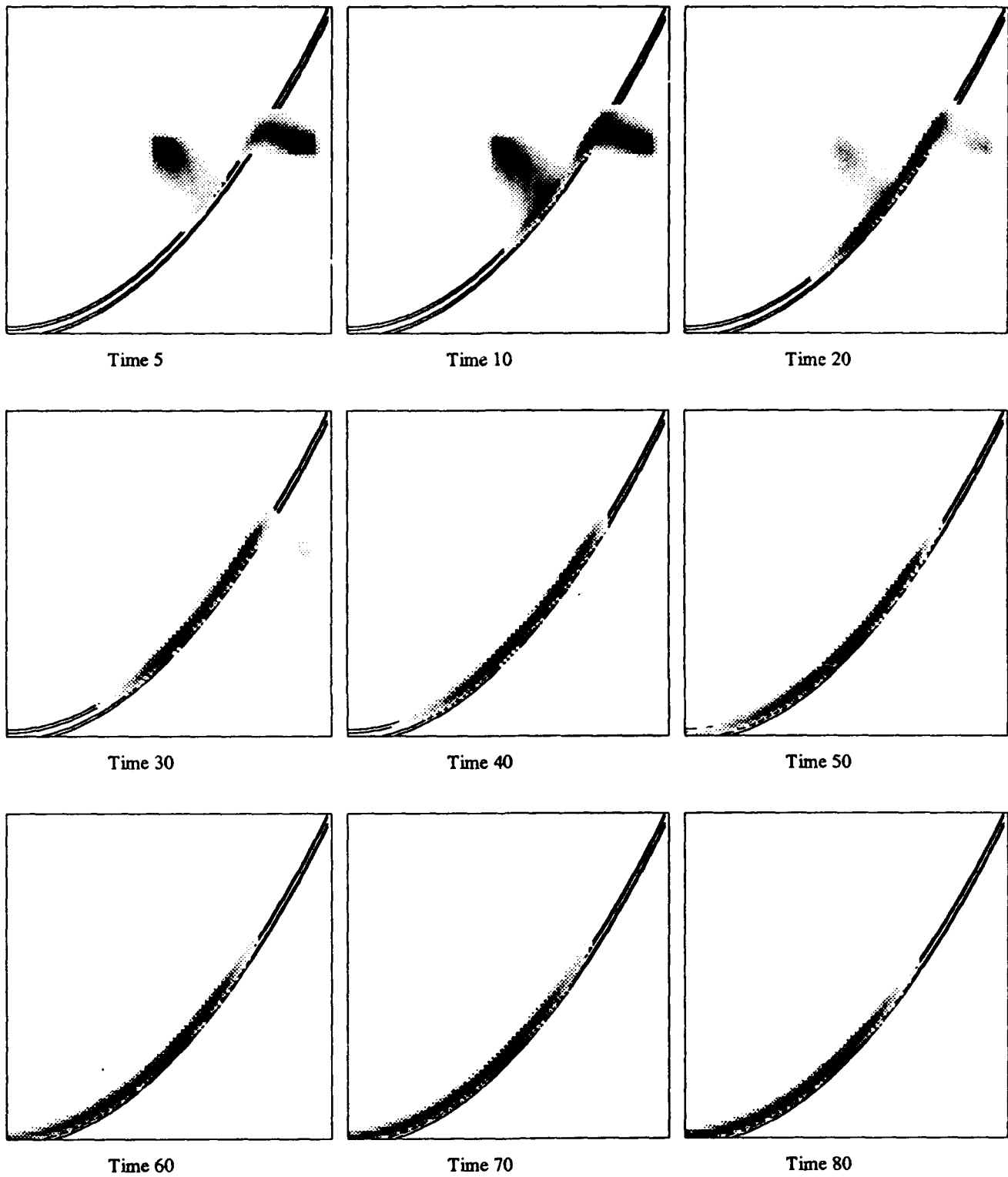


Figure 6. An Army Moving on the Path $p(x) = x^2$ with large σ .

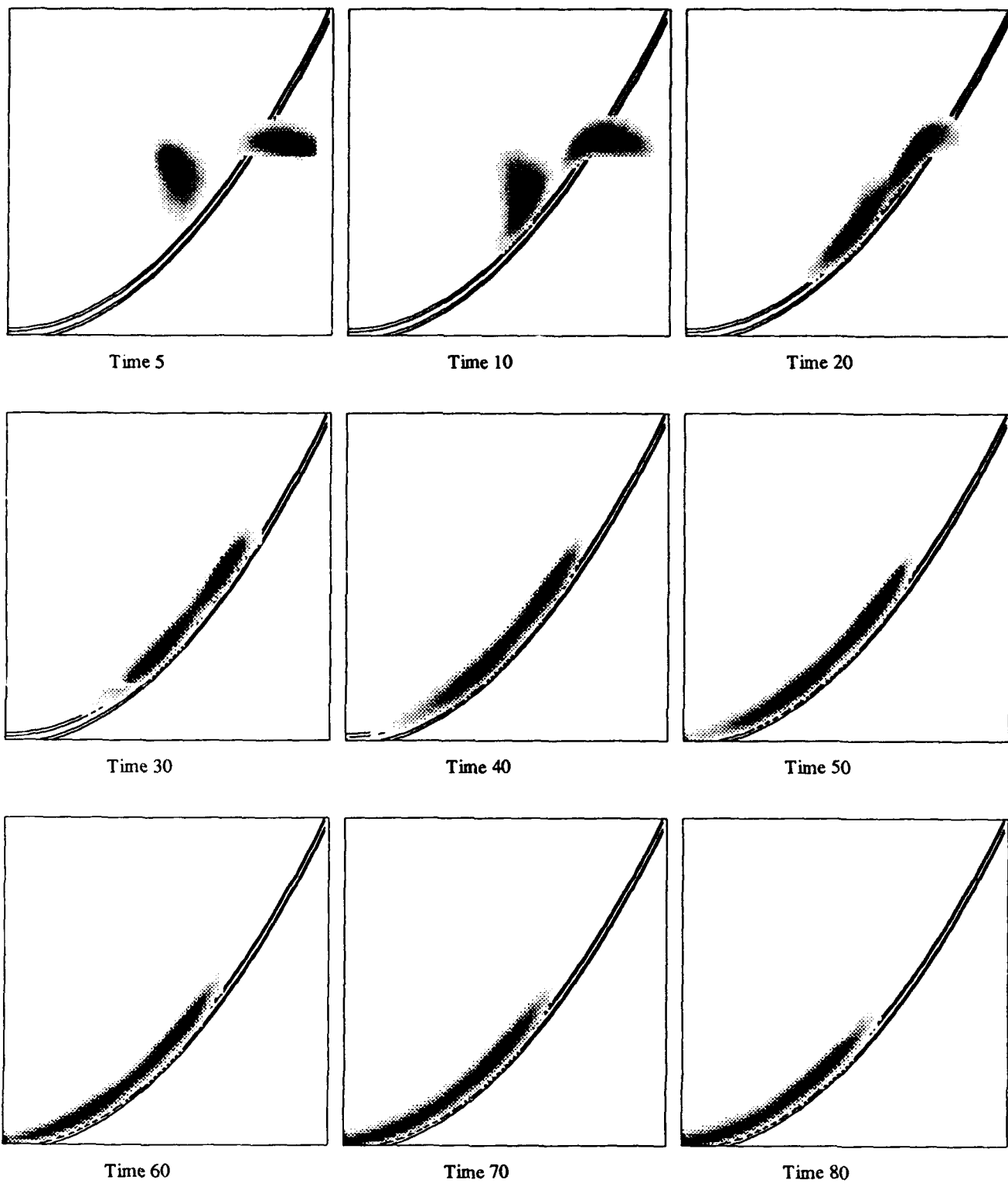


Figure 7. An Army Moving on the Path $p(x) = x^2$ with small σ .

large scale troop movement that requires an army to alter its original course of action in response to situations arising on the battlefield. The RDE model developed in the previous section will be extended to include automatic responsive movement. Consequently, no external action is required once the simulation starts.

We begin with a series of models in which two opposing forces, (blue and red), spread out to cover the battlefield until the opposing army is detected. Once detection occurs, the model assumes that the blue army will change its mission and move to contact the red army.

The simplest of these models is given by

$$b_t = D_1 b_{xx} + D_1 b_{yy} + f_x b_x + f_y b_y, \quad (5a)$$

$$r_t = D_2 r_{xx} + D_2 r_{yy}, \quad (5b)$$

with

$$f(x,y) = r(x,y). \quad (5c)$$

Diffusion is the only force affecting the distribution of the red army so its spatial distribution at any time during the simulation will be similar to a bivariate normal distribution. The dominant direction of movement for the blue army at a given time t , however, is determined by a velocity vector field that depends on the spatial distribution of the red army (provided $|\nabla f|$ is large relative to D_1). Notice that there are no interaction terms in these equations, so the armies do not fight. Figure 8 shows a sequence of snapshots generated by this simulation. The density function for the blue army is shown as shades of gray. Two contours of the red army distribution are also shown. In the early snapshots, the distribution for both armies is approximately bell shaped. As the simulation progresses, the blue army becomes strongly attracted to the center of the red army distribution. Eventually, as shown by the last snapshot, the whole blue army moves to the center of the red army distribution. Examining the vector field generated by $f(x,y) = r(x,y)$, the behavior of the blue army should not be unexpected since all the vectors in the field point to the maximum of the red army. Since armies generally do not sit on otherwise intact enemy positions, this model is of limited value as a battlefield simulation.

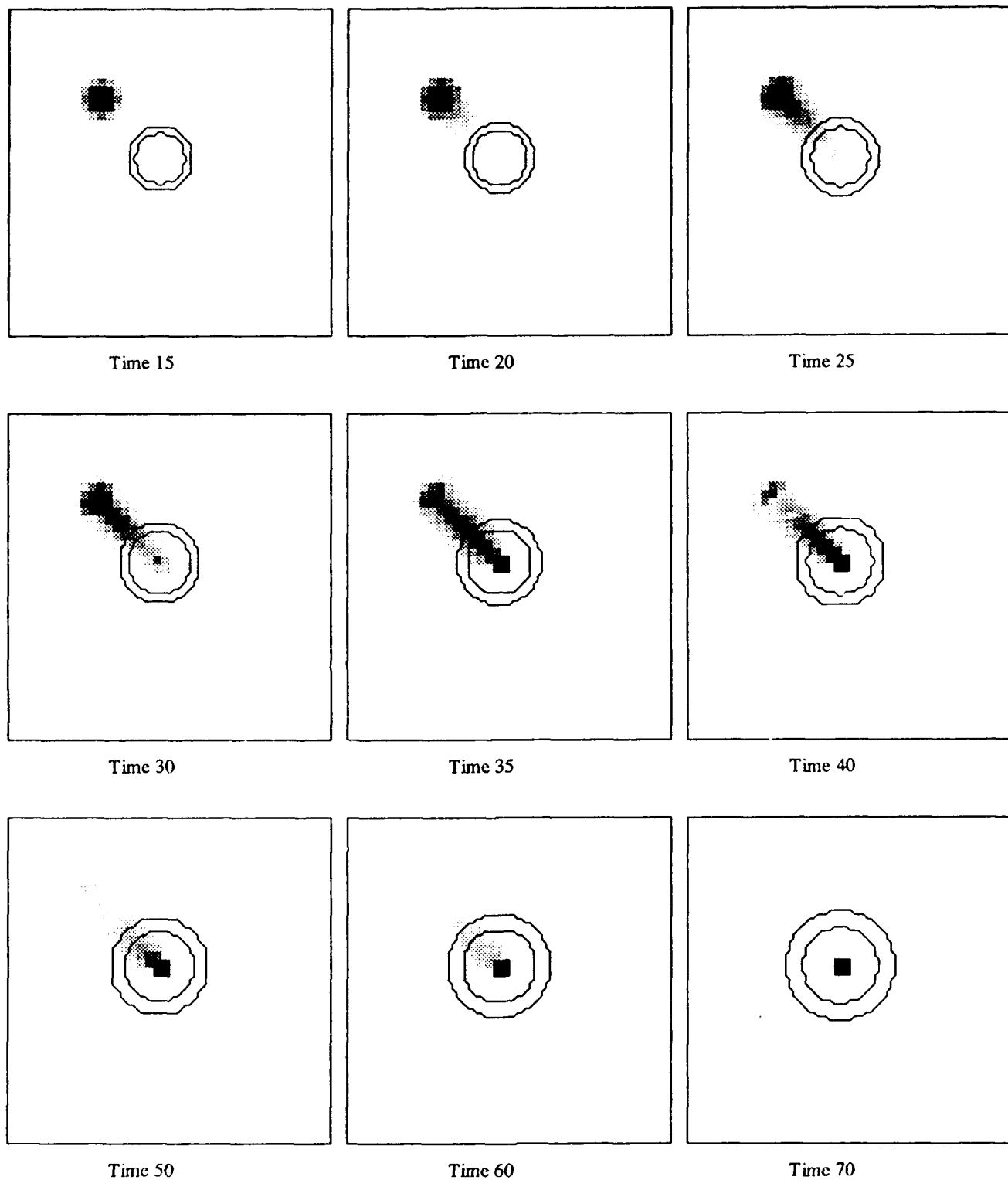


Figure 8. Uncontrolled Attraction of Blue Army to the Red Army.

In the next simulation, contact between the armies is limited to a battle area or front line between the armies. The velocity vector field of the last example must be modified so that the blue army is attracted to the edges of the red army distribution. Let f , the surface that determines the velocity vector field, be given by

$$f(x,y) = e^{-\eta(r(x,y) - r^*)^2}, \quad (6)$$

in which r^* is the density of the red army that is most "attractive" to the blue army, and η is a parameter that controls the width of the battle area. The model equations are the same as those given in Equations 5a and 5b. The term f is given by Equation 6. Figure 9 shows a sequence of snapshots generated by this simulation. The early snapshots are similar to those shown in Figure 8. In the later snapshots, a "front line" begins to form between the two armies. If the simulation is allowed to run long enough, the blue army will eventually surround the red army.

The next two simulations include interaction terms to simulate fighting in the model. Since these models are meant to be illustrative rather than accurate, only blue is allowed to fight. The equations for these models are

$$b_t = D_1 b_{xx} + D_1 b_{yy} + f_x b_x + f_y b_y - k_B r b, \quad (7a)$$

$$r_t = D_2 r_{xx} + D_2 r_{yy} + h_x b_x + h_y b_y - k_R r b, \quad (7b)$$

In the simulation shown in Figure 10, f is given by Equation 6, $k_B = 0$ and $h \equiv 0$. A "front line" between the armies gradually shrinks toward the center of the red distribution as red losses increase. In Figure 11, red is allowed to respond to blue by retreating, which we model by setting $h = -\omega b$ with ω a constant. This response is local in the sense that only red units "near" the blue force retreat. There is no global retreat of the red forces. Notice that retreat widens the battle area. It also, as expected, slows the rate of loss for the red army.

The last example in this section is a model of a more realistic engagement between a red and blue army. In this scenario, the blue army temporarily alters its original mission of marching along its road to attack a nearby red army. The attack breaks off when the red forces have suffered sufficient attrition, at which time, the blue forces return to

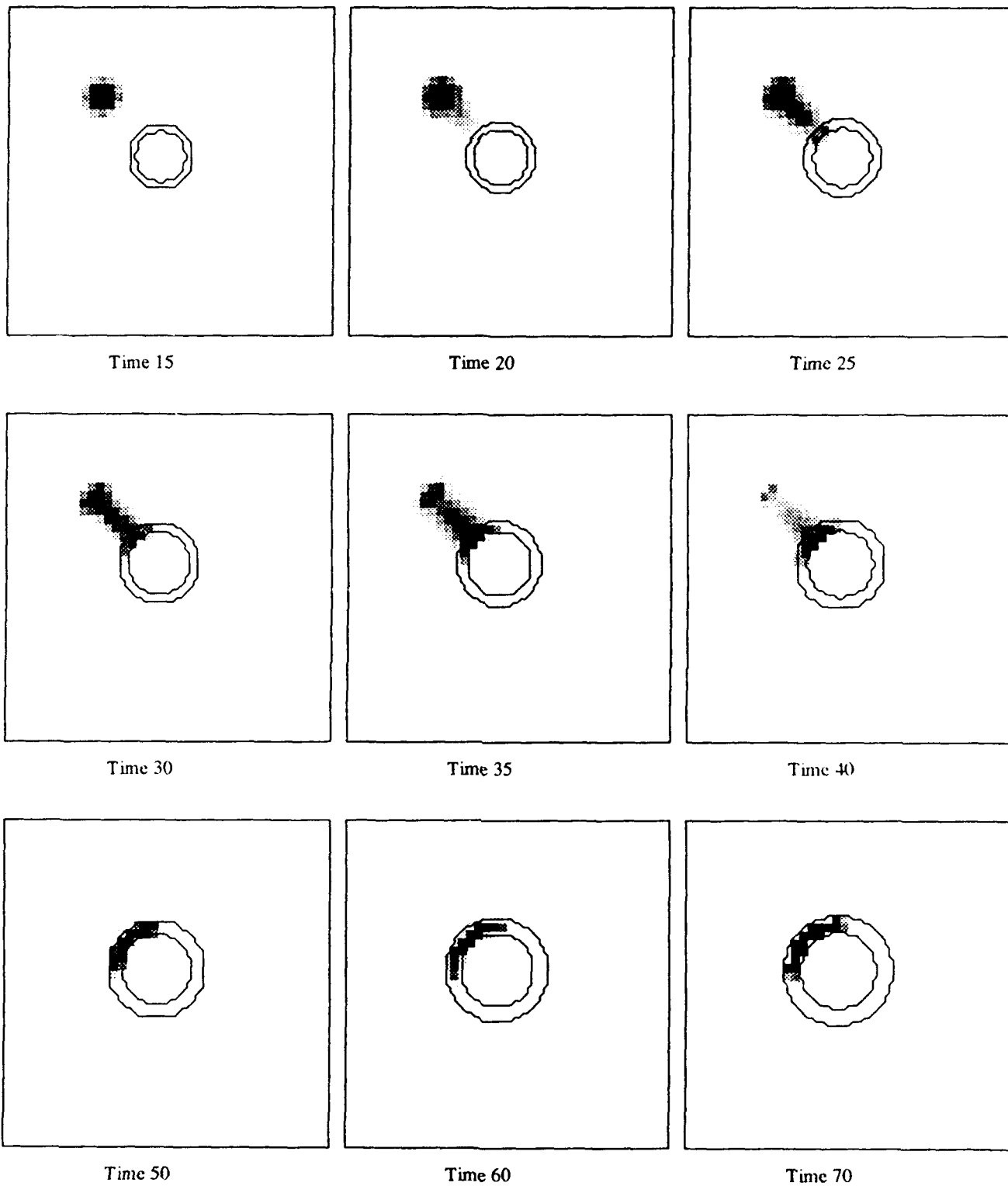


Figure 9. Controlled Attraction of Blue Army to the Red Army.

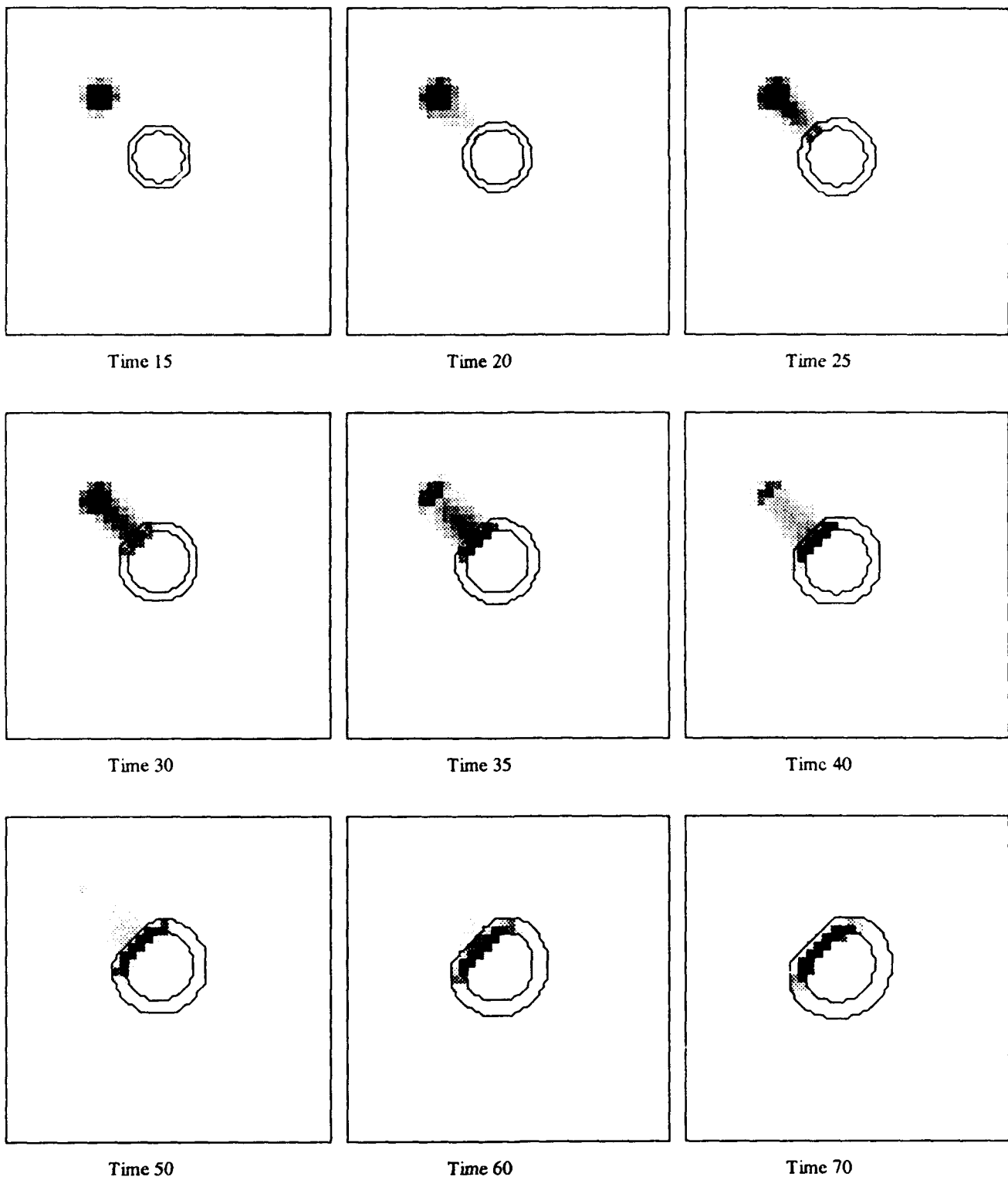


Figure 10. A Battle Between the Red and Blue Armies with No Response from Red.

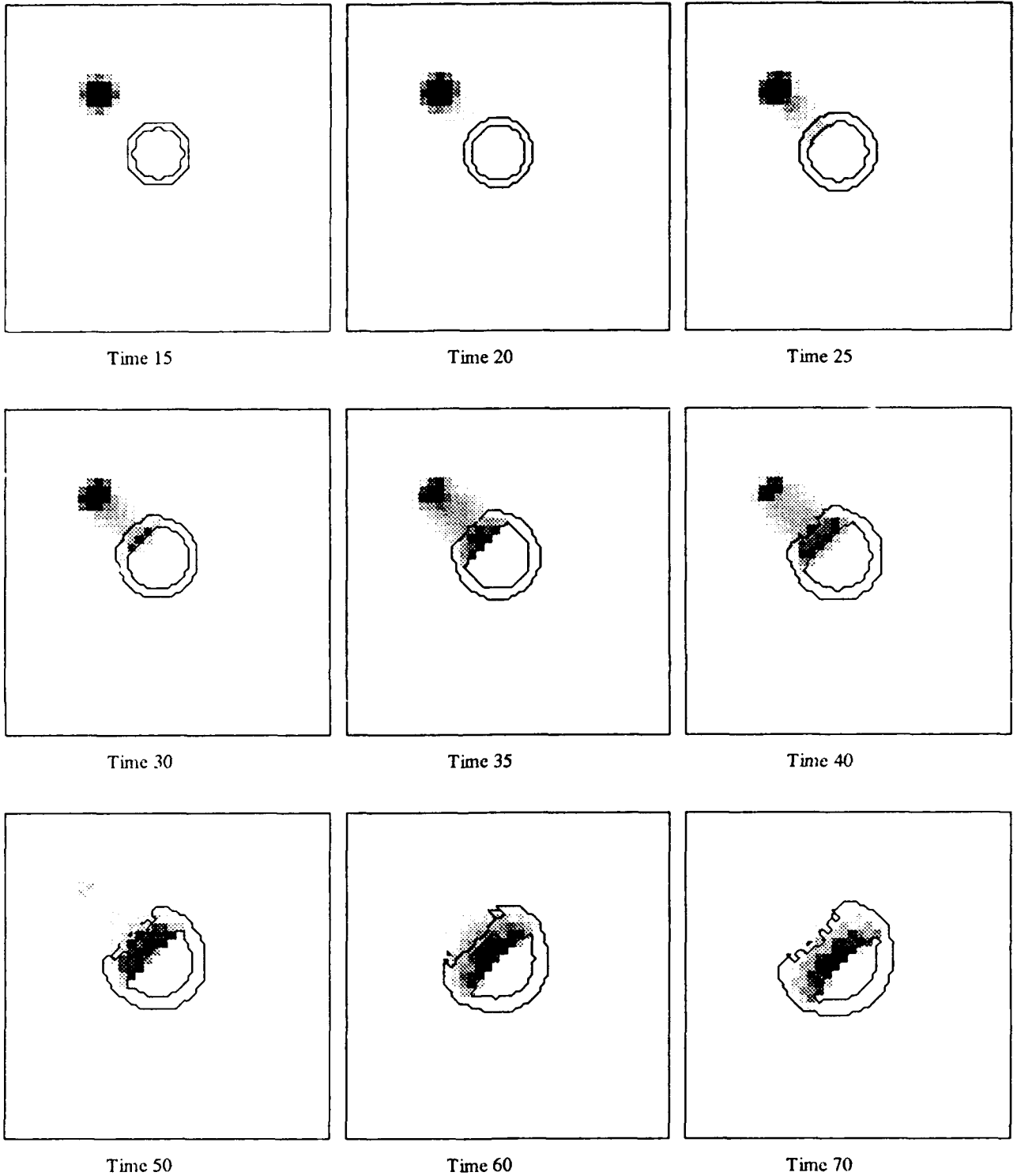


Figure 11. A Battle Between the Red and Blue Armies Allowing Red to Retreat.

their original mission. At any time, two vector fields affect the spatial distribution of the blue army. The first is determined by the road or path followed by the blue army. The second vector field is determined by the spatial distribution of the red army and the desired response of the blue army to the red army. Linearly combining these vector fields gives

$$f(x,y) = \gamma w(r(x,y)) + (1 - \gamma)g(x)e^{-\sigma(y - p(x))^2}, \quad (8a)$$

in which

$$w(r(x,y)) = e^{-\eta(r(x,y) - r^*)^2} \quad (8b)$$

in which r^* is the density of the red army that is most "attractive" to the blue army, and σ is a parameter that controls the width of the battle area, as defined earlier. The term, γ , determines the relative weight of the two vector fields. One very attractive feature of this model is that the user does not need *a priori* knowledge of where, when, or even if the red and blue armies will meet, nor is it necessary to know when to break off the engagement. If the concentration of the red army has been reduced enough, the blue army will return to its original battle plan.

Figure 12 shows a sequence of snapshots of an encounter between the red and blue armies. In the early snapshots, a portion of the blue army leaves the road to engage the red army. Fighting occurs along a "front line" between the red and blue armies. Both sides are affected by attrition. However, since $k_R \gg k_B$, blue has the advantage. Eventually, as illustrated in the last two snapshots, the red forces have been attrited sufficiently that the blue forces have begun to return to their original path.

5. CONCLUSIONS

This report illustrates that RDE models can be used to model large scale troop movement on the battlefield. The three types of movement described provide "building blocks", which can be used to model most movement on the battlefield. By combining these types of movement, it is possible to simulate realistic troop movement.

The proposed modeling method is not without its problems, however. The vector fields used in the tactical movement models must be constructed very carefully to achieve the desired results. It is particularly difficult to construct the responsive

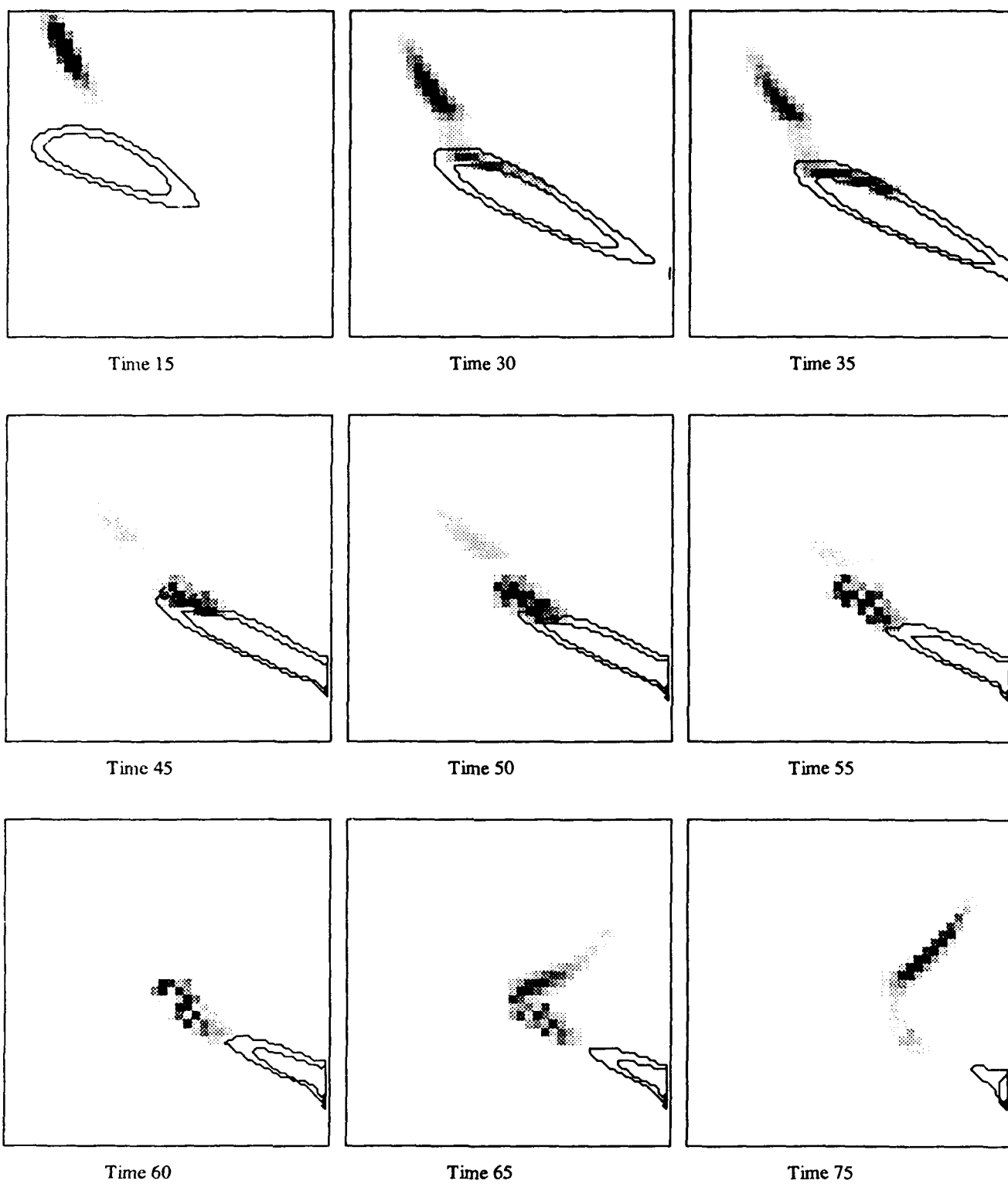


Figure 12. A Two-Sided Battle Between the Red and Blue Armies.

movement vector fields since these fields change dynamically with the movement of the enemy units. Additionally, the RDE model cannot run autonomously for the duration of a realistic battle. Command decisions may alter situations on the battlefield to such an extent that the RDE model will need to stop, and model parameters re-set then restarted. Scenarios for which RDE models are not appropriate exist, such as those that involve a single special purpose weapon.

We do not envision RDE models as total replacements for current methods. However, RDE models will be valuable tools in the field of combat simulation. The RDE models have relatively fast running computer codes that require small data sets. Treating the armies as masses reduces the amount of information needed since this information would be collected for armies rather than individual units. The RDE models provide more flexibility than current models since all possible maneuvers for the armies do not need to be defined initially; some contingency plans can be "built" into the equations. The behavior of the RDE models is easier to study since the qualitative theory of differential equations can be used to construct realistic models and to evaluate the sensitivity of the equations to the model parameters. Parametric studies are difficult and expensive to perform with current large scale battlefield simulation methods.

6. FUTURE WORK

Work on our first extension of this model has already begun. We will combine the RDE movement model with the Variable Resolution Terrain (VRT) model developed by Joe Wald and Carolyn Patterson of ARL (Wald and Patterson, 1992) which models the battlefield as a continuous surface. The surface created by the VRT model will be used to construct the velocity vector field for the RDE model.

In the future, this work will be expanded into a more complete battlefield simulation. To model more realistic tactics, we will need to add more armies, each with its own battle plan, and increase the complexity of the interaction functions. At the present time, the interaction functions require "contact" between opposing forces. These functions will be extended to model long range effects such as artillery and interactions such as intelligence and communication that require a "global" knowledge of opposing and friendly forces.

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